Optimization:
Optimal Pricing with Elasticity

Short Examples Series

using

Risk Simulator

File Name: *Optimization – Optimal Pricing with Elasticity*
Location: *Modeling Toolkit | Optimization | Optimal Pricing with Elasticity*
Brief Description: *This sample model illustrates price optimization that maximizes total revenues based on price elasticity of demand*
Requirements: *Modeling Toolkit, Risk Simulator*

This model is used to find the optimal pricing levels that will maximize revenues through the use of historical elasticity levels. The price elasticity of demand is a basic concept in microeconomics, which can be briefly described as the percentage change of quantity divided by the percentage change in prices. For example, if, in response to a 10 % fall in the price of a good, the quantity demanded increases by 20 %, the price elasticity of demand would be 20% / (−10 %) = −2. In general, a fall in the price of a good is expected to increase the quantity demanded, so the price elasticity of demand is negative but in some literature, the negative sign is omitted for simplicity (denoting only the absolute value of the elasticity). We can use this in several ways, including the point elasticity by taking the first derivative of the inverse of the demand function and multiplying it by the ratio of price to quantity at a particular point on the demand curve:

$$\varepsilon_d = \frac{\delta Q}{\delta P} \cdot \frac{P}{Q}$$

Where ε is the price elasticity of demand, P is price, and Q is quantity demanded.

Instead of using instantaneous point elasticities, this example uses the discrete version, where we define elasticity as:

$$\varepsilon_d = \frac{\%\Delta Q}{\%\Delta P} = \frac{Q_2 - Q_1}{\frac{Q_2 + Q_1}{2}} \div \frac{P_2 - P_1}{\frac{P_2 + P_1}{2}} = \frac{Q_2 - Q_1}{Q_2 + Q_1} \cdot \frac{P_2 + P_1}{P_2 - P_1}$$

To further simplify things, we assume that in a category of hotel rooms, cruise ship tickets, airline tickets, or any other products with various categories (e.g., standard room, executive room, suite, etc), there is an average price and average quantity of units sold per period. Therefore, we can further simplify the equation to:

$$\varepsilon_d = \frac{Q_2 - Q_1}{\overline{Q}} \div \frac{P_2 - P_1}{\overline{P}} = \frac{\overline{P}}{\overline{Q}} \cdot \frac{Q_2 - Q_1}{P_2 - P_1}$$

Where we now use the average price and average quantity demanded values $\overline{P}, \overline{Q}$.

If we have in each category the average price and average quantity sold, as in the model, we can compute the expected quantity sold given a new price if we have the historical elasticity of demand values. See Figure 1.

Historical Analysis

Type	Average Price Sold	Average Quantity Sold	Average Total Revenue
Single	$750	200	$150,000.00
Double	$812	180	$146,160.00
Deluxe	$865	150	$129,750.00
Executive	$1,085	100	$108,500.00
Premium Suite	$1,195	75	$89,625.00
Presidential	$1,458	50	$72,900.00

Figure 1: Sample historical pricing

In other words, if we take:

$$Q_1 - \varepsilon_d (P_2 - P_1) \frac{\overline{Q}}{\overline{P}}$$

we would get:

$$Q_1 - \left[\frac{\overline{P}}{\overline{Q}} \cdot \frac{Q_2 - Q_1}{P_2 - P_1} \right] (P_2 - P_1) \frac{\overline{Q}}{\overline{P}} = Q_2$$

To illustrate, suppose the price elasticity of demand for a single room during high season at a specific hotel property is 3.15 (we use the absolute value), where the average price last season was $750 and the average quantity of rooms sold was 200 units. What would happen if prices were to change from $750 (P1) to $800 (P2)? That is, what would happen to the quantity sold from 200 units (Q1)? See Figure 2.

Historical Analysis

Type	Average Price Sold	Average Quantity Sold	Average Total Revenue	Price Elasticity of Demand	Allocated New Price	Projected Quantity Sold
Single	$750	200	$150,000.00	3.15	$800.00	158
Double	$812	180	$146,160.00	2.85	$800.00	188
Deluxe	$865	150	$129,750.00	2.55	$1,000.00	90
Executive	$1,085	100	$108,500.00	2.35	$1,000.00	118
Premium Suite	$1,195	75	$89,625.00	1.65	$1,000.00	95
Presidential	$1,458	50	$72,900.00	1.45	$1,000.00	73

Figure 2: Elasticity simulation

Using the last equation, we compute the newly predicted quantity demanded at $800 per night to be:

$$Q_2 = Q_1 - \varepsilon_d (P_2 - P_1) \frac{\overline{Q}}{\overline{P}} = 200 - 3.15(800 - 750) \frac{200}{750} = 158$$

The higher the price, the lower the quantity demanded, and vice versa. Indeed, the entire demand curve can be reconstructed by applying different price levels. For instance, the demand curve for the Single room is reconstructed in Figure 3.
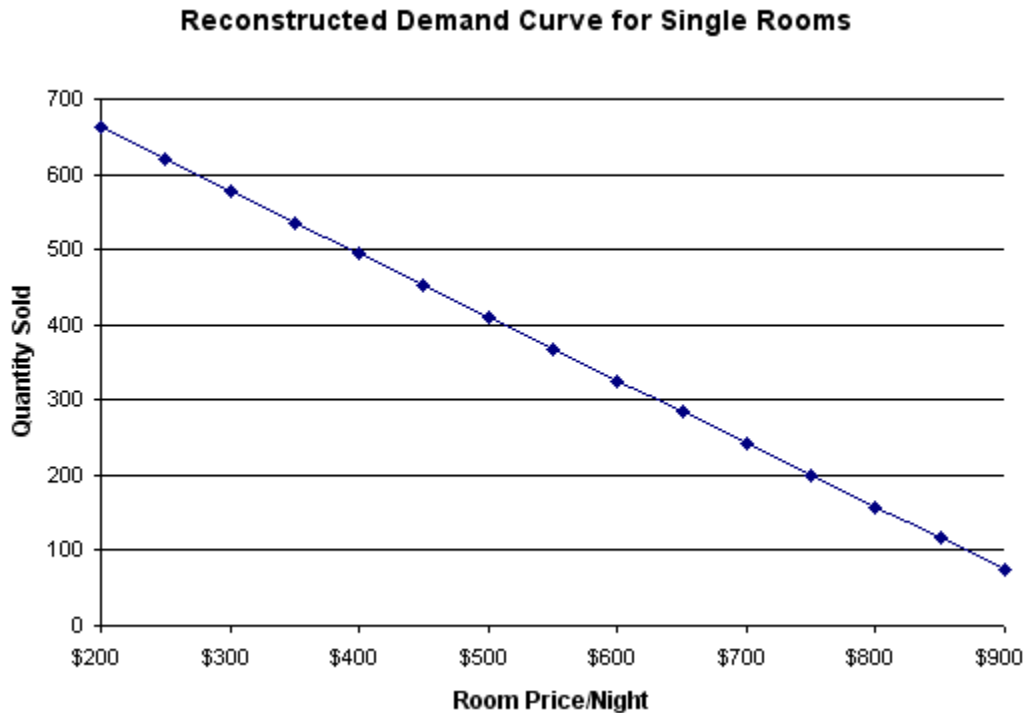

Figure 3: Reconstructed demand curve for a single room

Note: See the Simulation – Demand Curve and Elasticity Estimation model for examples on how to obtain elasticity measures.

Optimization Procedures

Using the principles of price elasticity of demand, we can now figure out the optimal pricing structure of these hotel rooms by setting:

Objective: Maximize Total Revenues

Constraints: Number of rooms available per type

Decision Variables: Price to charge for each type of room

This model already has the optimization set up. To run it directly, do the following:

1. Go to the *Model* worksheet and click on **Risk Simulator | Change Profile** and choose the *Optimal Pricing with Elasticity* profile.

2. Click on the **RUN OPTIMIZATION** icon or click on **Risk Simulator | Optimization | Run Optimization**.

3. Select the *Method* tab and select either **Static Optimization** if you wish to view the resulting optimal prices or **Stochastic Optimization** to run simulation with optimization multiple times, to obtain a range of optimal prices.

The results from a stochastic optimization routine are seen in the *Report* worksheet. In addition, several forecast charts will be visible once the stochastic optimization routine completes. For instance, looking at the Executive suites, select **Two-Tail**, type **90** in the Certainty box, and hit **TAB** on the keyboard to obtain the 90% confidence level (e.g., the optimal price to charge for the season is between $991 and $993 per night). See Figure 4.
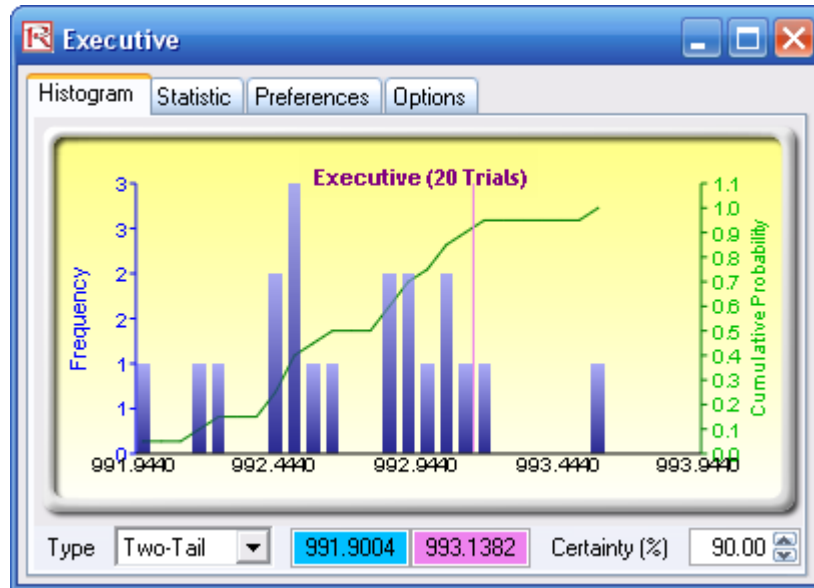

Figure 4: Distribution of stochastic optimization decision variable

To reset the model manually, do the following:

1. Go to the *Model* worksheet and click on **Risk Simulator | New Profile** and give the new profile a name.

2. *Reset* the values on prices. That is, enter 800 for cells H7, H8 and 1000 for cells H9 to H12. We do this so that we determine the initial starting prices that are easy to remember, versus the optimized price levels later on.

3. Set the objective. Select cell **J15** and click on the **O** (set objective) icon or click **Risk Simulator | Optimization | Set Objective**.

4. Set the decision variables. Select cell **H7** and click on the **D** (set decision variable) icon or click **Risk Simulator | Optimization | Set Decision**. Select **Continuous** and give it the relevant lower and upper bounds (see columns M and N) or click on the link icons and link the lower and upper bounds (cells M7 and N7).

5. Set the constraints. Click on the **C** icon or **Risk Simulator | Optimization | Set Constraints** and add the capacity constraints on the maximum number of available rooms (e.g., click **ADD** and link cell I7 and make it "<= 250" and so forth.

6. Click on the **RUN OPTIMIZATION** icon or click on **Risk Simulator | Optimization | Run Optimization**.

7. Select the *Method* tab and select either **Static Optimization** if you wish to view the resulting optimal prices, or **Stochastic Optimization** to run simulation with optimization multiple times, to obtain a range of optimal prices. But remember that in order to run a stochastic optimization procedure, you need to have assumptions set up. Select cell **G7** and click on **Risk Simulator | Set Input Assumption** and set an assumption of your choice or choose Normal distribution and use the default values. Repeat for cells G8 to G12, one at a time.